Individuals Tiebreaker 2023-2024

Student Name:

Team Name:

Rules and Directions

1. **DO NOT FLIP OR DETACH THIS PAGE UNTIL THE TIEBREAKER BEGINS.**
2. **Congratulations** for scoring well on the Individual Round! This is the CMM 2023-2024 **Individuals Tiebreaker**.
3. There are **4 questions with short-answers**, to be completed in **40 minutes** or **less**.
4. **You may not collaborate with your team** on the Individuals Tiebreaker. In addition, **no other collaboration, computers, calculators, or other outside aid is permitted**.
5. You are permitted to use ruler and compass but **not** a protractor.
6. On the **back side** of this page: write your answers in the corresponding boxes.
7. Answers must be *reasonably simplified* as described in the **CMM Conventions** document. You will not earn credit for a correct but unsimplified answer.
8. The time limit of this test is **40 minutes**, but you may submit answers at **any time** by handing this sheet of paper to the proctor. Students will be ranked based on the number of correct answers and (among those with the same number of correct answers) their submission time.

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Problems

Problem 1. Thomas the factory designer has two machines, S and P, designed for crafting electronic circuits. Electronic circuits take copper wires and iron plates to craft. Machine S crafts 1 circuit every second, consuming 4 copper wires and 3 iron plates per craft. Machine P crafts 1 circuit every 1.6 seconds, consuming 3 copper wires and 2 iron plates per craft.

Thomas has a supply of 18300 copper wires and an excess of iron plates. He must produce at least 5000 circuits to meet his manager's quota. He wants to produce these circuits as quickly as possible, but due to a power shortage, it is not possible for him to run both machines simultaneously. What is the minimum amount of time, in minutes, that it takes for him to produce the circuits needed? Express you answer as a decimal number rounded to the nearest 10th.

Problem 2. Six basketball teams participate in a single round-robin tournament. That is, each pair of teams play one game against each other, with one team winning and the other losing. The teams are all mutually equally matched: in any game, the chance each team has of winning is exactly $\frac{1}{2}$. Compute the probability that there are two teams that win the same number of games over the course of the tournament.

Problem 3. Find the number of ordered 6-tuples of integers

$$(x_1, x_2, x_3, y_1, y_2, y_3), \qquad \text{with } 0 \leq x_1, x_2, x_3, y_1, y_2, y_3 \leq 9,$$

such that $(x_1 - y_1) + 2(x_2 - y_2) + 3(x_3 - y_3)$ is divisible by 7.

Answer Submissions

Problem 1.

Problem 2.

Problem 3.