

CMM 2025 Integration Bee Finals

Caltech Math Meet

California Institute of Technology

January 25th, 2025

Please save questions for the end!



Guidelines

Logistics

Welcome to the CMM Integration Bee! We are very happy that you have decided to compete.

Please ensure that your whiteboard is clear and that you have two markers and an eraser. Please also make sure you can see the presentation clearly, as this will be the only method of seeing the integral.

I will be the announcer and operator of the slides. There are two people, one per board, who will be directly checking your answer as soon as you finish it. There will also be an observer in the case of a tie (time-wise) between the competitors.

For the audience, please do not attempt to help our contestants in any way, and we ask that you refrain from applause until after an elimination has been completed.

Logistics (continued)

The competition will proceed as follows for all the rounds. When an integral is shown, you will work silently to solve the integral as fast as possible. There is a mix of definite and indefinite integrals, so please put $+C$ when required!

Upon finishing the problem, please box your answer, verbally say **Done** (loud enough so that the board person next to you can hear it, but not loud enough that it would distract your competitor), and put your marker down. **No changes may be made to your answer once you say you are done.** At this point, the board person will check your boxed answer and simply say "**Correct**" or "**Incorrect**" loud enough for me to hear.

If your answer is correct, the round will end and you will be awarded a point. If incorrect, you must remain silent and let the other competitor either run out of time or answer. The time will be maintained by the observer. If there is a tie when the competitors finish the integral, the observer has **final say** in which the competitor answered first.

Structure

Here is the structure of the competition.

- The round of 16 and round of 8 will be scored as best of 1, and competitors will have 60 seconds to solve integrals in this round.
- The round of 4 will also be scored as best of 3, and competitors will have 90 seconds to solve integrals in this round.
- The round of 2 will be scored as best of 5, and competitors will have 120 seconds to solve integrals in this round.

The winner, or “Grand Integrator,” will be given a magical hat! With that, let’s begin!

Ready?

Please give an indication of when you are ready to start.

Easy

Integral 1

$$\int \sqrt{x} e^{\sqrt{x}} dx$$

Integral 1 Answer

$$e^{\sqrt{x}}(2x - 4\sqrt{x} + 4) + C$$

Integral 2

$$\int \sin(\sin(\cos(x))) \cos(\cos(x)) \sin(x) \, dx$$

Integral 2 Answer

$$\cos(\sin(\cos(x))) + C$$

Integral 3

$$\int \operatorname{arcsec}(\sqrt{x}) \, dx$$

Integral 3 Answer

$$x \operatorname{arcsec}(\sqrt{x}) - \sqrt{x-1} + C$$

Integral 4

$$\int_0^1 |1 - |1 - |1 - x||| \, dx$$

Integral 4 Answer

$$\frac{1}{2}$$

Integral 5

$$\int \frac{x^2 + 1}{1 - x} dx$$

Integral 5 Answer

$$-\frac{x^2}{2} - x - 2 \log |1 - x| + C$$

Integral 6

$$\int_1^{\pi/3} \sec(x) \left(\tan(x) \log(x) + \frac{1}{x} \right) dx$$

Integral 6 Answer

$$2 \log\left(\frac{\pi}{3}\right)$$

Integral 7

$$\int_0^1 \log^7\left(\frac{1}{x}\right) dx$$

Integral 7 Answer

5040

Integral 8

$$\int \frac{2x - 9\sqrt{x} + 9}{(x - 3\sqrt{x})^{1/3}} dx$$

Integral 8 Answer

$$\frac{6}{5} \cdot (x - 3\sqrt{x})^{5/3} + C$$

Integral 9

$$\int \frac{x^3 + 3x^2 + 3x}{x^4 + 4x^3 + 6x^2 + 4x + 1} dx$$

Integral 9 Answer

$$\ln|x+1| + \frac{1}{3(x+1)^3} + C$$

Integral 10

$$\int e^x \cdot \tan(e^x) dx$$

Integral 10 Answer

$$\ln |\sec e^x| + C$$

Integral 11

$$\int \sin(\arccos(x)) \, dx$$

Integral 11 Answer

$$\frac{x\sqrt{1-x^2} - \arccos(x)}{2} + C$$

Integral 12

$$\int \frac{3x + 2}{x^2 + 3x + 2} dx$$

Integral 12 Answer

$$4 \ln |x + 2| - \ln |x + 1| + C$$

Integral 13

$$\int (e^x - 1)^{-1} dx$$

Integral 13 Answer

$$\ln \left| \frac{1 - e^x}{e^x} \right| + C$$

Integral 14

$$\int \frac{1}{x \cdot \sqrt{x^{2025} - 1}} dx$$

Integral 14 Answer

$$\frac{2 \arctan(\sqrt{x^{2025} - 1})}{2025} + C$$

Integral 15

$$xe^x \cdot (\sin(x) + \cos(x)) dx$$

Integral 15 Answer

$$\frac{e(\sin(1) + \cos(1))}{2} - \frac{1}{2}$$

Integral 16

$$\int_0^1 (1 - x^{25})^{1/20} - (1 - x^{20})^{1/25} dx$$

Integral 16 Answer

0

Integral 17

$$\int \frac{1}{x + x^e} dx$$

Integral 17 Answer

$$\ln|x| - \frac{\ln(x^{e-1} + 1)}{e-1} + C$$

Integral 18

$$\int \frac{(x-1)e^x}{(x+1)^3} dx$$

Integral 18 Answer

$$\frac{e^x}{(x+1)^2} + C$$

Integral 19

$$\int \frac{\cos(4x) + 1}{\cot(x) - \tan(x)} dx$$

Integral 19 Answer

$$\frac{-\cos(4x)}{8} + C$$

Integral 20

$$\int \sec^2(x) \cdot \csc^3(x) \, dx$$

Integral 20 Answer

$$\tan(x) - 2 \cot(x) - \frac{\cot^3(x)}{3} + C$$

Integral 21

$$\int \frac{x + \sin(x)}{1 + \cos(x)} dx$$

Integral 21 Answer

$$\frac{x \tan(x)}{2} + C$$

Integral 22

$$\int \frac{1-x}{\sqrt{8-5x^2}} dx$$

Integral 22 Answer

$$\frac{\sqrt{8-5x^2}}{5} + \frac{1}{\sqrt{5}} \arcsin\left(\frac{1}{2}\sqrt{\frac{5}{2}}x\right) + C$$

Integral 23

$$\int \arctan \left(\sqrt{\frac{1 - \sin(x)}{1 + \sin(x)}} \right) dx$$

Integral 23 Answer

$$\frac{\pi x}{4} - \frac{x^2}{4} + C$$

Integral 24

$$\int_{-4}^4 e^{|x|} \cdot \{x\} dx$$

Integral 24 Answer

$$e^4 - 1$$

Integral 25

$$\int_{-\infty}^{\infty} \frac{e^{-x^2}}{1 + e^{x^{2025}} \cos(x)} dx$$

Integral 25 Answer

$$\frac{\sqrt{\pi}}{2}$$

Integral 26

$$\int_0^{\pi/2} \frac{\cos(x) - \sin(x)}{2025 - x^2 + \frac{\pi x}{2}} dx$$

Integral 26 Answer

0

Integral 27

$$\int_0^{2026\pi} \frac{1}{1 + e^{\sin(x)}} dx$$

Integral 27 Answer

$$1013\pi$$

Integral 28

$$\int \frac{(1 - \cos(x))^{3/10}}{(1 + \cos(x))^{13/10}} dx$$

Integral 28 Answer

$$\frac{5}{8} \cdot \left(\tan \left(\frac{x}{2} \right) \right)^{8/5} + C$$

Integral 29

$$\int_{-\pi/2}^{\pi/2} \left(x^2 + \ln \left(\frac{\pi - x}{\pi + x} \right) \right) \cdot \cos(x) \, dx$$

Integral 29 Answer

$$\frac{\pi^2}{2} - 4$$

Integral 30

$$\int \frac{1}{\cos^2(x) + 2025} dx$$

Integral 30 Answer

$$\sqrt{\frac{2}{13}} \cdot \pi$$

Medium

Integral 31

$$\int \frac{\sin(2x) - \sin^2 x}{\cos(2x) - \cos^2 x} dx$$

Integral 31 Answer

$$x - 2 \ln |\sin(x)| + C$$

Integral 32

$$\int \left(\sin(x) \ln(x) - \frac{\cos(x)}{x} \right) dx$$

Integral 32 Answer

$$-\cos(x) \ln(x) + C$$

Integral 33

$$\int \frac{x^2 - 1}{x^4 + 1}$$

Integral 33 Answer

$$\frac{1}{2\sqrt{2}} \cdot \ln \left| \frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1} \right| + C$$

Integral 34

$$\int \sqrt{\frac{\ln(x + \sqrt{1+x^2})}{1+x^2}} dx$$

Integral 34 Answer

$$\frac{2 \ln^{\frac{3}{2}} (\sqrt{x^2 + 1} + x)}{3} + C$$

Integral 35

$$\int \frac{1}{x(x^{2025} + 2)}$$

Integral 35 Answer

$$-\frac{1}{4050} \ln \left| \frac{2}{x^{2025}} + 1 \right| + C$$

Integral 36

$$\int \frac{1}{\sin^4 x + \cos^4 x} dx$$

Integral 36 Answer

$$\frac{\arctan(\sqrt{2} \tan(x) + 1) + \arctan(\sqrt{2} \tan(x) - 1)}{\sqrt{2}} + C$$

Integral 37

$$\int x \ln^2(x) dx$$

Integral 37 Answer

$$\frac{x^2 (2 \ln^2(x) - 2 \ln(x) + 1)}{4} + C$$

Integral 38

$$\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}$$

Integral 38 Answer

$$-6 \arctan(\sin(x)) + \sin(x) - 2 \csc(x) + C$$

Integral 39

$$\int \frac{1}{4 + 5 \sin x} dx$$

Integral 39 Answer

$$\frac{\ln \left| 2 \tan \left(\frac{x}{2} \right) + 1 \right| - \ln \left| \tan \left(\frac{x}{2} \right) + 2 \right|}{3} + C$$

Integral 40

$$\int_{\pi/6}^{\pi/3} \frac{\sin(x) + \cos(x)}{\sqrt{\sin(2x)}} dx$$

Integral 40 Answer

$$2 \arcsin \left(\frac{\sqrt{3} - 1}{2} \right)$$

Hard

Integral 41

$$\int -e^x \sqrt{4e^{2x} - 9} \, dx$$

Integral 41 Answer

$$\frac{9}{4} \ln \left| \frac{2}{3} e^x + \frac{1}{3} \sqrt{4e^{2x} - 9} \right| - \frac{1}{2} e^x \sqrt{4e^{2x} - 9} + C$$

Integral 42

$$\int \frac{\sqrt[3]{x}}{1+x} dx$$

Integral 42 Answer

$$3\sqrt[3]{x} - \ln|\sqrt[3]{x} + 1| + \frac{1}{2} \ln|x^{2/3} - \sqrt[3]{x} + 1| - \sqrt{3} \arctan\left(\frac{2}{\sqrt{3}}\sqrt[3]{x} - \frac{1}{\sqrt{3}}\right) + C$$

Integral 43

$$\int_0^{\infty} \frac{\sin \sqrt{3}x - \sin \frac{x}{\sqrt{3}}}{x} e^{-x^2} dx$$

Integral 43 Answer

$$\frac{\pi}{6}$$

Integral 44

$$\int_0^1 \frac{x^3}{(3 + 2x^2)^2} dx$$

Integral 44 Answer

$$\frac{1}{8} \ln\left(\frac{5}{3}\right) - \frac{1}{20}$$

Integral 45

$$\int_0^1 \arctan(x^2 - x + 1) \, dx$$

Integral 45 Answer

$$\ln(2)$$

Integral 46

$$\int_0^{\infty} e^{-x^2} \cos(2x) dx$$

Integral 46 Answer

$$\frac{\sqrt{\pi}}{2e}$$

Integral 47

$$\int \frac{1}{2 + 2 \sin(x) + \cos(x)} dx$$

Integral 47 Answer

$$\ln\left(\frac{\tan\left(\frac{x}{2}\right) + 1}{\tan\left(\frac{x}{2}\right) + 3}\right) + C$$

Integral 48

$$\int \prod_{k=1}^{n-1} \sqrt{2x^2 - 2x^2 \cos\left(\frac{2\pi k}{n}\right)} dx$$

Integral 48 Answer

$$x^n + C$$